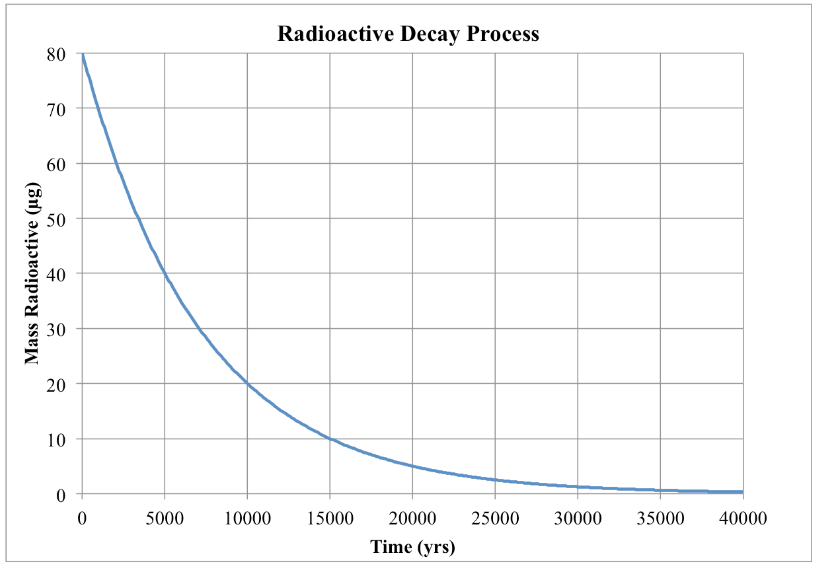
WKS
Half-Life

Name Key
Period ______ Date ______________

Questions 1-4 refer to the radioactive decay graph for a certain radioactive isotope seen at right.



1. What is the shape of the radioactive decay curve?
   This is an exponential decay curve.

2. What is the half-life of the isotope? Explain how you know.
   Half-life is 5000 years. At 5000 yr, half of the original sample is left, at 10,000 yrs half of that, etc.

3. How much of the isotope (in µg) remains after 15,000 years? What percentage of the original amount is this? How many half-lives have passed?
   After 15,000 years 10 µg remains. This is ⅛ or 12.5%, and 3 half-lives have passed.

4. How old is the sample when 2.5 µg remains? What percentage of the original amount is this? How many half lives have passed?
   The sample is 25,000 years old when there are 2.5 µg remaining. This is 3.1% or 1/32 of the original amount, and 5 half-lives have passed.

5. How is the half-life of a radioisotope is similar to a sporting tournament in which the losing team is eliminated?
   In each round of the tournament, half the teams are eliminated

6. $^{24}_{11}Na$ undergoes β-decay with a half-life of 15 hours. If we start with 64 grams of $^{24}_{11}Na$, fill in the chart for how much $^{24}_{11}Na$ we will have after each time interval.

Time	0 h	15 h	30 h	45 h	60 h	75 h	90 h	105 h
Mass of $^{24}_{11}$Na	64 g	32 g	16 g	8 g	4 g	2 g	1 g	0.5 g

Label half-lives 1 2 3 4 5 6 7

7. $^{259}_{103}Lr$ undergoes α-decay with a half-life of 6.2 seconds. If we begin with 80 µg of $^{259}_{103}Lr$, how long will it take before only 5 µg remains? 24.8 seconds

Time	0 s	6.2 s	12.4 s	18.6 s	24.8 s	31 s	37.2 s	43.4 s
Mass of $^{257}_{103}$Lr	80 µg	40 µg	20 µg	10 µg	5 µg	2.5 µg	1.25 µg	0.625 µg

Label half-lives 1 2 3 4 5 6 7

8. The half-life of protactinium-234, which undergoes β-decay, is 6.75 hours. What percentage of a given sample will remain after 27 hours? [Start with 100%]

$$n = \frac{\text{Time}}{t_{1/2}} = \frac{27 \text{ hrs}}{6.75 \text{ hrs}} = 4 \text{ half-lives; } A = (100\%)\left(\tfrac{1}{2}\right)^4 = (100\%)\left(\tfrac{1}{16}\right) = \boxed{6.25\%}$$

(or) $\begin{matrix}0 \text{ hrs}\\100\%\end{matrix} \xrightarrow[\times\frac{1}{2}]{+6.75 \text{ hrs}} \begin{matrix}6.75 \text{ hrs}\\50\%\end{matrix} \xrightarrow[\times\frac{1}{2}]{+6.75 \text{ hrs}} \begin{matrix}13.50 \text{ hrs}\\25\%\end{matrix} \xrightarrow[\times\frac{1}{2}]{+6.75 \text{ hrs}} \begin{matrix}20.25 \text{ hrs}\\12.5\%\end{matrix} \xrightarrow[\times\frac{1}{2}]{+6.75 \text{ hrs}} \begin{matrix}27 \text{ hrs}\\\boxed{6.25\%}\end{matrix}$

9. A rock once contained 1.0 mg of uranium-238, but now contains only 0.25 mg. Given that the half-life for uranium-238 is 4.5×10^9 years, how old is the rock?

$$0.25 \text{ mg} = (1.0 \text{ mg})\left(\tfrac{1}{2}\right)^n \Rightarrow \tfrac{0.25 \text{ mg}}{1.0 \text{ mg}} = \left(\tfrac{1}{2}\right)^n = \tfrac{1}{4}; \; n = 2 \text{ half-lives; Time} = n \cdot t_{1/2} = 2\,(4.5\times10^9 \text{yrs}) = \boxed{9.0\times10^9 \text{yrs}}$$

(or) $\begin{matrix}0 \text{ yrs}\\1.0 \text{ mg}\end{matrix} \xrightarrow[\times\frac{1}{2}]{4.5\times10^9 \text{ yrs}} \begin{matrix}4.5\times10^9 \text{ yrs}\\0.50 \text{ mg}\end{matrix} \xrightarrow[\times\frac{1}{2}]{4.5\times10^9 \text{ yrs}} \begin{matrix}\boxed{9.0\times10^9 \text{ yrs}}\\0.25 \text{ mg}\end{matrix}$

Use the following chart to answer questions 5-7…

Radioactive Isotope	Approximate half-life	Decay Mode
Radon-222	4 days	α
Iodine-131	8 days	β
Carbon-14	5,730 years	β
Plutonium-239	24,120 years	α

10. If we start with 20 g of plutonium-239, how much would remain after 48,240 years?

$$n = \frac{\text{Time}}{t_{1/2}} = \frac{48{,}240 \text{ yrs}}{24{,}120 \text{ yrs}} = 2 \text{ half-lives; } A = (20 \text{ g})\left(\tfrac{1}{2}\right)^2 = (20 \text{ g})\left(\tfrac{1}{4}\right) = \boxed{5 \text{ g}}$$

(or) $\begin{matrix}0 \text{ yrs}\\20 \text{ g}\end{matrix} \xrightarrow[\times\frac{1}{2}]{24{,}120 \text{ yrs}} \begin{matrix}24{,}120 \text{ yrs}\\10 \text{ g}\end{matrix} \xrightarrow[\times\frac{1}{2}]{424{,}120 \text{ yrs}} \begin{matrix}28{,}240 \text{ yrs}\\\boxed{5 \text{ g}}\end{matrix}$

11. If 3.75 g of carbon-14 remains after 22,920 years, how much was initially present?

$$n = \frac{\text{Time}}{t_{1/2}} = \frac{22{,}920 \text{ yrs}}{5{,}730 \text{ yrs}} = 4 \text{ half-lives; } 3.75 \text{ g} = A_0\left(\tfrac{1}{2}\right)^4 = A_0\left(\tfrac{1}{16}\right) \Rightarrow A_0 = (3.75 \text{ g})(16) = \boxed{60.0 \text{ g}}$$

(or) $\begin{matrix}0 \text{ yrs}\\\boxed{60.0 \text{ g}}\end{matrix} \xleftarrow[\div\frac{1}{2}]{-5730 \text{ yrs}} \begin{matrix}5730 \text{ yrs}\\30.0 \text{ g}\end{matrix} \xleftarrow[\div\frac{1}{2}]{-5730 \text{ yrs}} \begin{matrix}11{,}460 \text{ yrs}\\15.0 \text{ g}\end{matrix} \xleftarrow[\div\frac{1}{2}]{-5730 \text{ yrs}} \begin{matrix}17{,}190 \text{ yrs}\\7.50 \text{ g}\end{matrix} \xleftarrow[\div\frac{1}{2}]{-5730 \text{ yrs}} \begin{matrix}22{,}920 \text{ yrs}\\3.75 \text{ g}\end{matrix}$

12. If 24 μg of iodine-131 has decayed to 0.75 μg, how much time has passed?

$$0.75 \text{ μg} = (24 \text{ μg})\left(\tfrac{1}{2}\right)^n \Rightarrow \tfrac{0.75 \text{ μg}}{24 \text{ μg}} = \left(\tfrac{1}{2}\right)^n = \tfrac{1}{32} \Rightarrow n = 5 \text{ half-lives; } 5 = \frac{\text{Time}}{t_{1/2}} = \frac{\text{Time}}{8 \text{ days}}; \text{ Time} = 5\times8 \text{ days} = \boxed{40 \text{ days}}$$

(or) $\begin{matrix}0 \text{ days}\\24 \text{ μg}\end{matrix} \xrightarrow[\times\frac{1}{2}]{+8 \text{ days}} \begin{matrix}8 \text{ days}\\12 \text{ μg}\end{matrix} \xrightarrow[\times\frac{1}{2}]{+8 \text{ days}} \begin{matrix}16 \text{ days}\\6.0 \text{ μg}\end{matrix} \xrightarrow[\times\frac{1}{2}]{+8 \text{ days}} \begin{matrix}24 \text{ days}\\3.0 \text{ μg}\end{matrix} \xrightarrow[\times\frac{1}{2}]{+8 \text{ days}} \begin{matrix}32 \text{ days}\\1.5 \text{ μg}\end{matrix} \xrightarrow[\times\frac{1}{2}]{+8 \text{ days}} \begin{matrix}\boxed{40 \text{ days}}\\0.75 \text{ μg}\end{matrix}$