Significant Figure Rules

A. Significant Figures in Measurements

In any measurement, there is some level of error or uncertainty. Scientists have devised a way to indicate the level of precision in any number, called Significant Figures. The more precise a number is, the more digits you can write. How can you tell from a number how many "Sig Figs" there are in it? There is a simple process for determining this:

- Start from left and find the first *non-zero* digit (*). This is the first (most) significant digit.
- Count digits as you move to the right, starting with the most significant digit
  - If there is a decimal point in the number, count to the last digit (◊), *even if it is a 0*.
  - The last digit you counted is the last (least) significant digit; the count is the number of significant digits

    3.78 cm : 3 sig figs   0.00002001 mg : 4 sig figs   0.095380 s : 5 sig figs

    52,884.00 cm : 7 sig figs 6.109×10^{6} L : 4 sig figs 7.3000×10^{-3} J : 5 sig figs

    - note: zeroes to the left of the first significant digit are placeholders to establish the *magnitude* of a number. 0.00002001 mg is NOT the same as 2001 mg.
    - note 2: for numbers written in scientific notation, all digits in the coefficient are significant.
  - If there is no decimal point, count to the last *non-zero* digit

    200 km : 1 sig fig   34,500 kg : 3 sig figs   9,000,000 g : 1 sig fig

    655,030 s : 5 sig figs   50,040 L : 4 sig figs   703 V : 3 sig figs

    - Note: zeroes at the end of a number are also placeholders: 34,500 kg is not the same as 345 kg. If you want to make zeroes at the end of a number significant, you must use scientific notation. For example, to express 34,500 kg to 4 significant figures, you must write 3.450×10^{4} kg, not 3450 kg, which is not the same value.
- Exact numbers have unlimited (infinite) significant figures. These numbers are not made by measuring, but include the following types:
  - Counts–numbers obtained by enumeration. These are always integer values and contain no uncertainty. E.g. 25 students, 100 pencils, 345 steps.
  - Defined numbers (conversion factors). These are considered exact even if there is no decimal point present. E.g. 2.54 cm/in, 100 cm/m, 60 min/hr.

B. Significant Figures in Calculations

Any calculation is only as precise as the numbers going into it. You many have to "round off" any results to a certain precision.

1. Rounding off: Look at the digit following the last significant digit (◊)
   a. If the digit is less than 5, drop it and everything following it (round down)
   b. If the digit is greater than 5, round the last significant digit up one and drop everything following it (round up)
   c. If the digit equals 5, round the last significant digit to the closest EVEN numeral
   d. If the number is < 1, keep all leading 0'; they are placeholders
   e. If the number is > 1 and you have removed any non-zero digits, replace them with a 0 to act as a placeholder

Examples: Round the following number to 3 significant figures:

77.0653 g $\rightarrow$ 77.1 g	6,300,178.2 cm^3 $\rightarrow 6.30\times10^6$ cm^3
0.00023350 mL $\rightarrow$ 0.00234 mL	10.2030 ft $\rightarrow$ 10.2 ft
2.895×10^{21} km $\rightarrow 2.90\times10^{21}$ km	759.9 ns $\rightarrow$ 760. ns or 7.60×10^2 ns

2. In multiplication and division, the answer has the same number of significant figures as the input with the LEAST number of significant figures. The result can be no more precise than the least precise number used to calculate it.
Examples:
$4.32 \text{ mm} \times 1.7 \text{ mm} = 7.344 \text{ mm} \rightarrow 7.3 \text{ mm}$ (two sig. figs.)
$38.742 \text{ g} \div 0.421 \text{ g} = 92.0238 \rightarrow 92.0$ (three sig. figs.)
$5.40 \text{ m} \times 3.21 \text{ m} \times 1.871 \text{ m} = 32.4319 \text{ m}^3 \rightarrow 32.4 \text{ m}^3$ (three sig. figs.)

3. In addition and subtraction, the answer goes to the same place (tens, hundredths, etc.) as the LEAST precise input number (whose least significant digit is farthest left).
Examples:

87 g	853.1 L	7532 m
1543 g	– 627.443 L	+ 400 m
+ 19 g	225.657 L $\rightarrow$ 225.7 L	7932 m $\rightarrow$ 7900 m
4313 g $\rightarrow$ 43 g		

Are Significant Figures Important? A Fable

A student once needed a cube of metal which had to have a mass of 83 grams. He knew the density of this metal was 8.67 g/mL, which told him the cube's volume. Believing significant figures were invented just to make life difficult for chemistry students and had no practical use in the real world, he calculated the volume of the cube as 9.573 mL. He thus determined that the edge of the cube had to be 2.097 cm. He took his plans to the machine shop where his friend had the same type of work done the previous year. The shop foreman said, "Yes, we can make this according to your specifications—but it will be expensive."

"That's OK," replied the student. "It's important." He knew his friend has paid $35, and he had been given $50 out of the school's research budget to get the job done.

He returned the next day, expecting the job to be done. "Sorry," said the foreman. "We're still working on it. Try next week." Finally the day came, and our friend got his cube. It looked very, very smooth and shiny and beautiful in its velvet case. Seeing it, our hero had a premonition of disaster and became a bit nervous. But he summoned up enough courage to ask for the bill. "$500, and cheap at the price. We had a terrific job getting it right -- had to make three before we got one right."

"But—but—my friend paid only $35 for the same thing!"

"No. He wanted a cube 2.1 cm on an edge, and your specifications called for 2.097 cm. We had yours roughed out to 2.1 cm that very afternoon, but it was the precision grinding and lapping to get it down to 2.097 cm which took so long and cost the big money. The first one we made was 2.089 cm on one edge when we got finished, so we had to scrap it. The second was closer, but still not what you specified. That's why the three tries."

"Oh!"