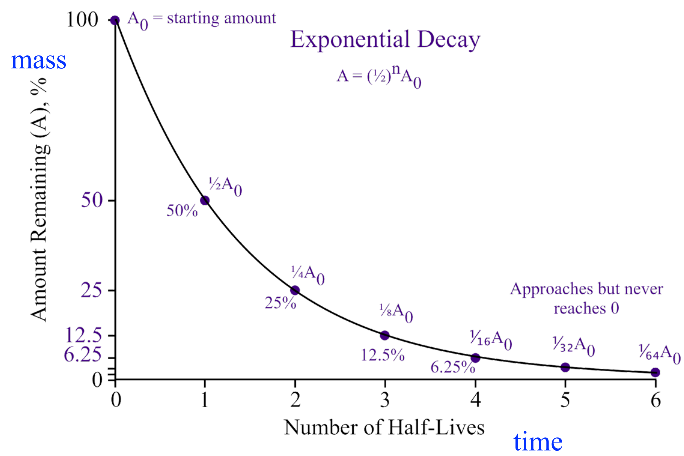
Notes 5-3: Radioactive Decay Rates, Half Life and Radiochemical Dating

- Half-life: $t_{1/2}$ = time for ½ of any radioactive isotope to undergo decay.
  - Shorter half-life means more rapid decay, higher radioactivity
    - Each isotope has unique ½-life.
  - After each half-life, half of the previous amount remains: ½, ¼, ⅛, 1⁄16, 1⁄32, etc.
    - Graph of amount vs. time (at right)
      - *Exponential* decay: Y decreases, X is an exponent in the equation:
      - $A = A_0(½)^n$, where A = final amount, A_0 = initial amount, n = # of half lives = elapsed time/$t_{1/2}$
      - Must solve either time or mass first to find n



  - **First type of problem: given time and $t_{1/2}$, use A or A_0 to find the other (time first)**
    - The half-life of radium-226 is 1600 years. How many grams of a 0.25g sample will remain after 4800 years?
      - Find n=4800 yrs/1600 yrs = 3, A_0 = 0.25 g; A = (0.25 g) $(½)^3$ = 0.031 g
      - Or solve visually: 0.25 g → 0.125 g → 0.0625 g → 0.03125 g = 0.031 g (Each arrow = one ½-life)
    - The half-life of thorium-227 is 18.72 days. How many grams were initially present if 10.0 g remain after 37.44 days?
      - $n = 37.44 \text{ days}/18.72 \text{ days} = 2;\ 10.0 \text{ g} = A_0(\frac{1}{2})^2$ so $A_0 = (10.0 \text{ g})(2)^2 = 40.0 \text{ g}$
      - Or work backwards: 10.0 g → 20.0 g → 40.0 g
  - **Second type of problem: know A and A_0, use time or $t_{1/2}$ to find the other (mass first)**
    - A bone contains 12.5% of the original amount of C-14 ($t_{1/2}$ = 5730 yr). How old is the bone?
      - Know A=12.5%, A_0=100%, first find n: 12.5% = (100%)$(½)^n$ ⟹ n=3 half lives
        - ♦ Or: 100% → 50% → 25% → 12.5%, or 3 ½-lives
      - n=3 half lives, so time = n × $t_{1/2}$ = 3×5730 yrs = 17200 yrs (elapsed time = age)
  - Isotopic dating using C-14
    - C-14 constantly formed in atmosphere:
      $^{14}_{7}N + ^{1}_{0}n \rightarrow ^{14}_{6}C + ^{1}_{1}p$
      - Absorbed by living organisms (C cycle)
      - Decays at steady rate:
        $^{14}_{6}C \rightarrow ^{14}_{7}N + ^{0}_{-1}e$
      - Level stays constant while organism is alive (about 1 in 1×10^{12} C atoms)
        - ♦ Steady state reached: rate of formation = rate of tecay
    - Decreases after death (decaying but no longer absorbed)
      - can give age to about 50,000 years
    - Use isotopes with longer half-lives for longer times (e.g. U-238 $t_{1/2}=4.5\times10^9$ yr) or non-organic materials
      - Well-known decay series