

**Writing out  $K_{sp}$  expressions:** A  $K_{sp}$  expression is just a specific type of equilibrium expression.  $K_{sp}$  expressions are only used for relatively insoluble solids in water. Every saturated solution must satisfy its  $K_{sp}$  expression because every saturated solution is at equilibrium. (*Ion concentrations are constant because rate of dissolving = rate of crystallization.*)

- 1) Write the balanced chemical equations for the following solids dissolving in water. Then, write the  $K_{sp}$  expression for each. *Remember: Pure solids are never written in equilibrium expressions. Don't forget exponents.*



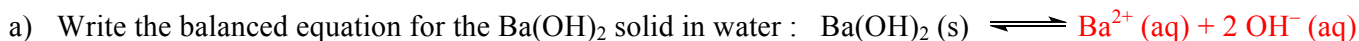
$$K_{sp} = [\text{Sb}^{3+}]^2[\text{S}^{2-}]^3$$



$$K_{sp} = [\text{Mn}^{2+}][\text{CO}_3^{2-}]$$

**First Type of  $K_{sp}$  Calculation: Determining a  $K_{sp}$  value from concentrations.**

- 2) A sample of  $\text{Ba}(\text{OH})_2(\text{s})$  is added to pure water and allowed to come to equilibrium. A saturated solution forms. The concentration of  $\text{Ba}^{2+}(\text{aq})$  in this saturated solution is 0.108 M. What is  $K_{sp}$  of  $\text{Ba}(\text{OH})_2$ ?

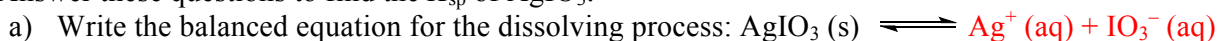


b) Since  $[\text{Ba}^{2+}] = 0.108 \text{ M}$ . What must be the concentration of the  $\text{OH}^{-}$  ions? (*Use balanced eq.*) 0.216 M

c) Write the  $K_{sp}$  expression (no numbers.) Then, plug in  $[\text{Ba}^{2+}]$  and  $[\text{OH}^{-}]$  values and solve for the  $K_{sp}$ .

$$K_{sp} = [\text{Ba}^{2+}][\text{OH}^{-}]^2 = (0.108)(0.216)^2 = \boxed{0.00504 = 5.04 \times 10^{-3}}$$

- 3) A sample of  $\text{AgIO}_3(\text{s})$  is dissolved in water and a saturated solution is formed. The  $[\text{IO}_3^{-}] = 1.7 \times 10^{-4} \text{ M}$ . Answer these questions to find the  $K_{sp}$  of  $\text{AgIO}_3$ .



b) What is  $[\text{Ag}^{+}]$ ?  $1.7 \times 10^{-4} \text{ M}$

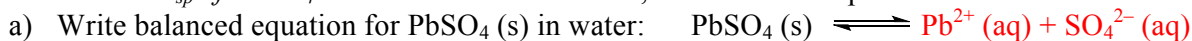
c) Write the  $K_{sp}$  expression (no numbers.) Then, plug in  $[\text{Ag}^{+}]$  and  $[\text{IO}_3^{-}]$  values and solve for the  $K_{sp}$ .

$$K_{sp} = [\text{Ag}^{+}][\text{IO}_3^{-}] = (1.7 \times 10^{-4})(1.7 \times 10^{-4}) = \boxed{2.9 \times 10^{-8}}$$

**Second type of  $K_{sp}$  calculations: Solving for an unknown ion concentration in a saturated solution.**

- 4) Determine the equilibrium concentrations of  $\text{Pb}^{2+}$  and  $\text{SO}_4^{2-}$  in a saturated solution of  $\text{PbSO}_4$ .

*Given:  $K_{sp}$  of  $\text{PbSO}_4 = 1.3 \times 10^{-8}$*  To do so, follow these steps:



b) Suppose  $[\text{Pb}^{2+}] = x$ . Then,  $[\text{SO}_4^{2-}] = \underline{x}$

c) Write  $K_{sp}$  expression and then plug in  $K_{sp}$  value and concentration values in terms of "x's". Solve for x.

$$K_{sp} = [\text{Pb}^{2+}][\text{SO}_4^{2-}] = x \cdot x = x^2 = 1.3 \times 10^{-8}; x = \sqrt{1.3 \times 10^{-8}} = \boxed{1.1 \times 10^{-4} \text{ M}}$$

d) Thus,  $[\text{Pb}^{2+}] = \underline{1.1 \times 10^{-4} \text{ M}}$  and  $[\text{SO}_4^{2-}] = \underline{1.1 \times 10^{-4} \text{ M}}$

5) Follow these steps to find the equilibrium concentrations of  $\text{Ni}^{2+}$  and  $\text{OH}^-$  in a saturated solution of  $\text{Ni}(\text{OH})_2$ ?

Given:  $K_{sp}$  of  $\text{Ni}(\text{OH})_2 = 1.6 \times 10^{-16}$

a) Write balanced equation for  $\text{Ni}(\text{OH})_2$  solid in water :  $\text{Ni}(\text{OH})_2 (\text{s}) \rightleftharpoons \text{Ni}^{2+} (\text{aq}) + 2 \text{OH}^- (\text{aq})$

b) Suppose  $[\text{Ni}^{2+}] = x$ . Then,  $[\text{OH}^-] = 2x$

c) Write  $K_{sp}$  expression and then plug in  $K_{sp}$  value and concentration values in terms of "x's". Solve for x.

$$K_{sp} = [\text{Ni}^{2+}][\text{OH}^-]^2 = (x)(2x)^2 = 4x^3 = 1.6 \times 10^{-16}$$

$$x = \sqrt[3]{\frac{1.6 \times 10^{-16}}{4}} = 3.4 \times 10^{-6} \text{ M}$$

d) Thus,  $[\text{Ni}^{2+}] = 3.4 \times 10^{-6} \text{ M}$  and  $[\text{OH}^-] = 6.8 \times 10^{-6} \text{ M}$

**Third type of  $K_{sp}$  calculation: Determining if precipitate will form. Solve for Q. (You're not at equil.)**

6) A solution is prepared by dissolving  $9.2 \times 10^{-4}$  moles of  $\text{PbCl}_2 (\text{s})$  into 100. mL of hot water. Should a precipitate form if the solution is cooled to  $25^\circ\text{C}$ ? (Assume it is cooled too quickly for a supersaturated solution to form). Given:  $K_{sp}$  of  $\text{PbCl}_2$  at  $25^\circ\text{C} = 1.6 \times 10^{-5}$ . To answer this question, follow these steps:

a) Write the balanced equation for  $\text{PbCl}_2 (\text{s})$  in water:  $\text{PbCl}_2 (\text{s}) \rightleftharpoons \text{Pb}^{2+} (\text{aq}) + 2 \text{Cl}^- (\text{aq})$

b) Write the Q expression-- no numbers. (same as  $K_{sp}$  expression -- you are just not at equilibrium)

$$Q = [\text{Pb}^{2+}][\text{Cl}^-]^2$$

c) Determine the concentrations of the ions in the hot solution.

$$[\text{Pb}^{2+}] = 9.2 \times 10^{-4} \text{ mol}/0.100 \text{ L} = 9.2 \times 10^{-3} \text{ M}; [\text{Cl}^-] = 2 [\text{Pb}^{2+}] = 1.8 \times 10^{-2} \text{ M}$$

d) Plug in concentrations and solve for Q.

$$Q = (9.2 \times 10^{-3})(1.8 \times 10^{-2})^2 = 3.0 \times 10^{-6}$$

e) Q is (greater than, **less than**) the  $K_{sp}$ .

f) Thus, there are (more, **fewer**) ions dissolved in the hot solution than at the saturation point at  $25^\circ\text{C}$ .

g) Thus, if the solution is cooled to  $25^\circ\text{C}$  would solid precipitate? No (Assume one would disturb the solution if it were supersaturated.) Thus, the resulting solution at  $25^\circ\text{C}$  is (**unsaturated**, saturated).

7) Suppose  $7.5 \times 10^{-13}$  moles of  $\text{Bi}(\text{NO}_3)_3$  solid and  $1.3 \times 10^{-13}$  moles of  $\text{Na}_2\text{S}$  solid are added to 500 mL of water at  $25^\circ\text{C}$ . Does  $\text{Bi}_2\text{S}_3$  solid precipitate? ( $K_{sp}$  of  $\text{Bi}_2\text{S}_3 = 1.1 \times 10^{-73}$ ) To answer, follow these steps:

a) What is the initial concentration of  $\text{Bi}^{3+}$  ions in the 500 mL solution? (assume no reaction yet)

$$[\text{Bi}^{3+}] = [\text{Bi}(\text{NO}_3)_3] = 7.5 \times 10^{-13} \text{ mol}/0.500 \text{ L} = 1.5 \times 10^{-12} \text{ M}$$

b) initial  $[\text{NO}_3^-]$ ?

$$= 3 [\text{Bi}(\text{NO}_3)_3] = 4.5 \times 10^{-12} \text{ M}$$

initial  $[\text{Na}^+]$ ?

$$= 2 [\text{Na}_2\text{S}] = 2.6 \times 10^{-13} \text{ M}$$

initial  $[\text{S}^{2-}]$ ?

$$= [\text{Na}_2\text{S}] = 1.3 \times 10^{-13} \text{ mol}/0.500 \text{ L} = 2.6 \times 10^{-13} \text{ M}$$

c) Write the balanced equation for  $\text{Bi}_2\text{S}_3$  solid in water:  $\text{Bi}_2\text{S}_3 (\text{s}) \rightleftharpoons 2 \text{Bi}^{3+} (\text{aq}) + 3 \text{S}^{2-} (\text{aq})$

d) Write the Q expression-- no numbers. (same as  $K_{sp}$  expression -- you are just not at equilibrium)

$$Q = [\text{Bi}^{3+}]^2 [\text{S}^{2-}]^3$$

e) Plug in the initial concentrations of the ions involved and solve for Q.

$$Q = (1.5 \times 10^{-12})^2 (2.6 \times 10^{-13})^3 = 4.0 \times 10^{-62}$$

f) Q is (greater than, **less than**) the  $K_{sp}$ . Thus, there are (more, **fewer**) ions than at saturation pt.

g) Would a precipitate form? Yes The resulting solution is (**unsaturated**, saturated)